WEEKLY TEST OYJ TEST - 19
SOLUTION Date 25-08-2019

[PHYSICS]

1.

As, $E_P I_P = P_i$ (Input Power)

or $I_P = \frac{P_i}{E_P} = \frac{4000}{100} = 40$ amp.

2.

Given: No. of turns across primary coil, $N_p = 50$

Number of turns across secondary coil, $N_s = 1500$

Magnetic flux linked with primary coil, $\phi = \phi_0 + 4t$

$\therefore$ Voltage across the primary coil,

$$V_p = \frac{d\phi}{dt} = \frac{d}{dt}(\phi_0 + 4t) = 4 \text{ volt}$$

Also, $$\frac{V_s}{V_p} = \frac{N_s}{N_p}$$

$\therefore$ $$V_s = \left(\frac{1500}{50}\right) \times 4 = 120 \text{ V}.$$

3.

The net magnetic flux through a closed surface, will be zero.

$\oint B ds = 0$ because there are no magnetic monopoles.

4.

The induced emf e in the secondary coil is given by:

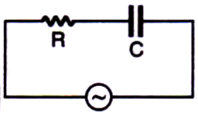
$$e = -\frac{d\phi}{dt} = -M\frac{dI}{dt}$$

or $$|e| = M\frac{dI}{dt}$$

$\therefore$ $$|e| = 5 \times \frac{10}{5 \times 10^{-4}} = 1 \times 10^5 \text{ V}.$$

5.

$$I = \frac{V}{\sqrt{R^2 + (1/c\omega)^2}}$$

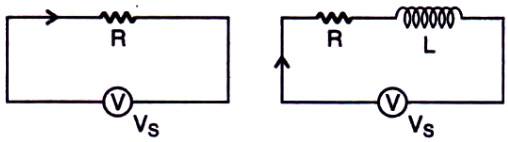


$$V_c = \frac{V}{\sqrt{\left[R^2 + \left(\frac{1}{c\omega}\right)^2\right]}} \times \left(\frac{1}{c\omega}\right)$$

$$= \frac{V}{\sqrt{(Rc\omega)^2 + 1}}$$

If we fill a dielectric material, then c gets increased and therefore V_c gets decreased. Hence, correct option is (a).

6.



$$P = I_{rms}^2 R = \left(\frac{V_s}{R}\right)^2 R = \frac{V_s^2}{R}$$

$$P' = I_{rms}^2 R = \left(\frac{V_s}{Z}\right)^2 R$$

$$= \left(\frac{V_s^2}{R}\right)\left(\frac{R}{Z}\right)^2 = P\left(\frac{R}{Z}\right)^2$$

7.

The current through the coil 1 is

$$I_1 = I_0 \sin \omega t$$

where I_0 is the peak value of current.

Magnetic flux linked with the coil 2 is

$$\phi_2 = MI_1 = MI_0 \sin \omega t$$

where M is the mutual inductance between the two coils.

The magnitude of induced emf in coil 2 is

$$|e_2| = \frac{d\phi_2}{dt} = \frac{d}{dt} = (MI_0 \sin \omega t)$$

$$= MI_0 \omega \cos \omega t$$

$\therefore$ Peak value of voltage induced in the coil 2 is

$$= MI_0 \omega = 150 \times 10^{-3} \times 2 \times 2\pi \times 50$$

$$= 30\,\pi \text{ volt.}$$

8.

$$V = V_0 \quad \text{for} \quad 0 \le t \le \frac{T}{2}$$

$$V = 0 \quad \text{for} \quad \frac{T}{2} \le t \le T$$

$$V_{rms} = \left[\frac{\int_0^T V^2 dt}{\int_0^T dt}\right]^{1/2} = \left[\frac{\int_0^{T/2} V_0^2 dt + \int_{T/2}^{T} (0) dt}{\int_0^T dt}\right]^{1/2}$$

$$= \left[\frac{V_0^2}{T}[t]_0^{T/2}\right]^{1/2} = \left[\frac{V_0^2}{T}\left(\frac{T}{2}\right)\right]^{1/2} = \left[\frac{V_0^2}{2}\right]^{1/2}$$

$$V_{rms} = \frac{V_0}{\sqrt{2}}.$$

9.

For series L-C-R circuit,

$$V = \sqrt{V_R^2 + (V_L - V_C)^2}$$

$$= \sqrt{(80)^2 + (40-100)^2} = 100 \text{ V}.$$

10.

11.

Amount of magnetic flux linked with inductor is,

$$\phi = Li$$

Now, the emf induced in the inductor is given by:

$$e = -\frac{d\phi}{dt} = -\frac{d}{dt}(Li) = -L\frac{di}{dt}$$

or $$|e| = L\frac{di}{dt}$$

Here, induced current, $\frac{V}{R} = \frac{10}{5} = 2 \text{ A}$

Circuit switches off in 1 millisecond

or $$dt = 1 \times 10^{-3} \text{ s}$$

and $$L = 10 \text{ H}$$

$\therefore$ Induced emf in inductor is,

$$|e| = 10 \times \frac{2}{1 \times 10^{-3}} = 2 \times 10^4 \text{ V}.$$

12.

$$Q = \frac{\omega L}{R} = \frac{1}{R} \times \frac{1}{\sqrt{LC}} \times L = \frac{1}{R}\sqrt{\frac{L}{C}}$$

$$= \frac{1}{3} \times \sqrt{\frac{1}{9}} = \frac{1}{9}.$$

13.

Compare $I = I_0 \sin(\omega t + \phi)$ with the given equation.

Then, we get;

$$I_0 = \sqrt{2}$$

$\therefore$ $$I_{rms} = I_0/\sqrt{2} = \sqrt{2}/\sqrt{2} = 1 \text{ A}.$$

14.

Phase difference between E and $I = \theta$

$\therefore$ $$\tan\theta = \frac{X_L - X_C}{R}$$

Now, $$X_L = 2\pi f L$$

$$= 2\pi \times 50 \times \left(\frac{200}{\pi} \times 10^{-3}\right)$$

$$= 20\ \Omega$$

$$X_C = \frac{1}{2\pi f C}$$

$$= \frac{1}{2\pi \times 50} \times \frac{\pi}{10^{-3}} = 10\ \Omega$$

$$R = 10\ \Omega$$

$\therefore$ $$\tan\theta = \frac{20-10}{10} = \frac{10}{10} = 1 = \tan\frac{\pi}{4}$$

$\therefore$ $$\theta = \pi/4$$

The current will lag by $\pi/4$.

15.

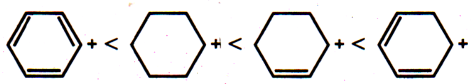
[CHEMISTRY]

16. (*c*) : Fluoroethane is prepared by halogen exchange method

$$2\ C_2H_5Cl + Hg_2F_2 \xrightarrow{\Delta} 2\ C_2H_5F + Hg_2Cl_2$$

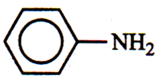
......Swarts reaction

17. (*b*) : The reactivity depends upon the stability of the carbocations which the alkyl chlorides give on ionization. Now stability of the carbocations increases in the order:



Therefore, the reactivity of their corresponding alkyl chlorides follows the same order, *i.e.*, IV < I < II < III.

18. (*a*) : Diazonium salts react with CuBr/HBr (Sandmeyer reaction) to form bromobenzene.

19. (*a*) :  $\xrightarrow[298\,K]{NaNO_2/HCl}$

$$C_6H_5-\overset{+}{N}\equiv NCl^- \xrightarrow[-HCl]{HBF_4} \left[C_6H_5-\overset{+}{N}\equiv NBF_4^-\right]$$

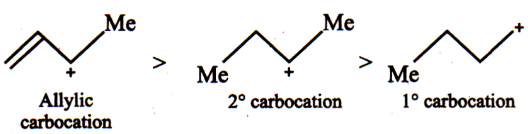
Benzenediazonium chloride (A)

$$\xrightarrow[-N_2,-BF_3]{\Delta} C_6H_5F$$

Fluorobenzene

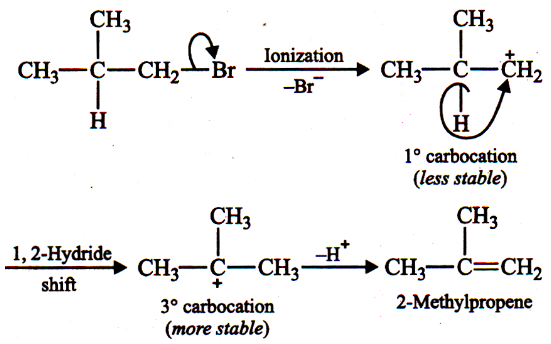
20. (*d*) : Boiling points increase as the number of carbon atoms in the alkyl halide increases. Therefore, the boiling points of II, III and IV with four carbon atoms is higher than that of I with three carbon atoms. Further, the boiling points decrease with the number of branches. Thus, the boiling points of the four alkyl halides increase in the order: I < IV < III < II.

21. (*c*) : The reactivity in S_N1 reactions depends upon the stability of the intermediate carbocations. Since the stability of the carbocations derived from (*A*), (*B*) and (*C*) follows the order:



therefore, S_N1 reactivity of these three bromides follows the order: *B* > *C* > *A*, *i.e.*, option (*c*) is correct.

22. (*c*) : Although the given alkyl halide is 1°, yet the carbocation which it generates on ionization, rearranges to form a more stable 3° carbocation which in presence of a strong base (CH_3O^-) prefers to undergo elimination rather than substitution to afford 2-methylpropene



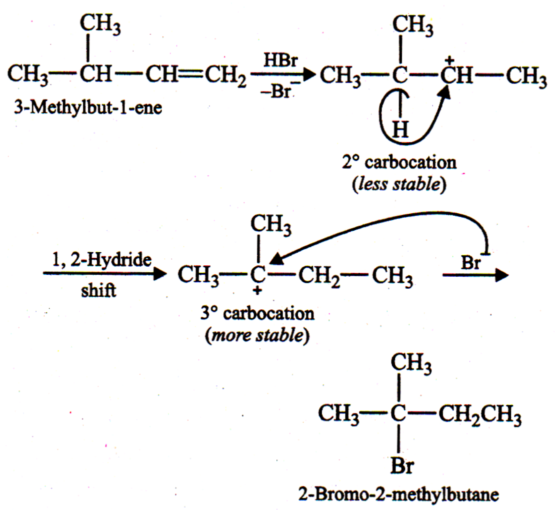
23. (*a*) : As the steric hindrance increases at the α-carbon atom holding the halogen, the reactivity towards S_N2 mechaism decreases, *i.e.*, I > III > II > IV.

24 (*c*) : Electron-withdrawing groups increase and electron-donating groups decrease the reactivity towards nucleophilic substitution. Further, more the number of electron-withdrawing groups, higher is the reactivity. Thus, option (*c*), *i.e.*, *iv* > *iii* > *ii* < > *i* is correct.

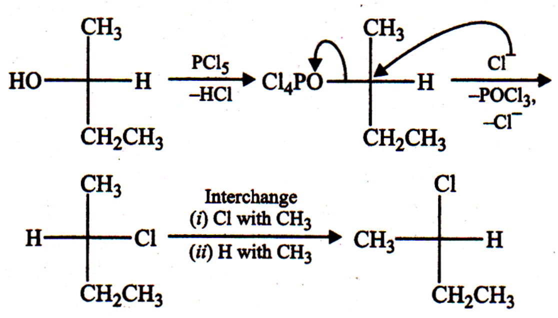
25 (*c*) : $\underset{(X)}{CH_2 = CH_2} \xrightarrow{HBr} CH_3CH_2Br \xrightarrow{Aq.\,NaOH}$

$$CH_3CH_2OH \xrightarrow[\text{(Iodoform reaction)}]{Na_2CO_3/\text{excess } I_2} CHI_3$$

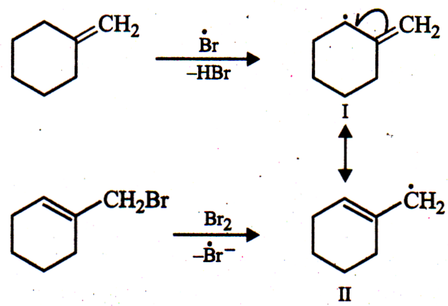
26. (*d*)



27. (*a*) : PCl_5 reacts by S_N2 mechanism and hence inversion of configuration occurs.



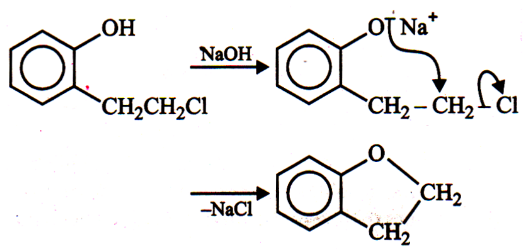
28. (*c*) : The mechanism of allylic bromination is



Since endocyclic (within ring) double bond is more stable than exocyclic (outside ring) double bond, therefore, initially formed less stable free radical (I) gets converted into the more stable free radical (II) which then reacts with Br_2 to give the product.

29. (*b*) : The dipole moment of CH_3Cl, CH_3Br and CH_3I decreases as the electronegativity of the halogen decreases. However, due to much smaller size of F as compared to that of Cl, dipole moment (product of charge and distance) of CH_3F is lower than that of CH_3Cl. Thus, option (*b*) is correct.

30. (*c*) :



[MATHEMATICS]

31.

(a) Let $I = \int_0^{\pi/2} \log(\tan x)dx$

then,

$$I = \int_0^{\pi/2} \log\left[\tan\left(\frac{\pi}{2} - x\right)\right]dx \quad \because \int_0^a f(x)dx = \int_0^a f(a-x)dx$$

or $I = \int_0^{\pi/2} \log(\cot x)dx$

$$= \int_0^{\pi/2} \log\left(\frac{1}{\tan x}\right)dx$$

$$= -\int_0^{\pi/2} \log \tan x \, dx = -I$$

$\therefore \; I = -I$ or $2I = 0$ or $I = 0$.

32.

(c) Let $I = \int_0^{\pi} \frac{x \tan x}{(\sec x + \tan x)}dx$ ...(*i*)

then, $I = \int_0^{\pi} \frac{(\pi - x)\tan(\pi - x)}{[\sec(\pi - x) + \tan(\pi - x)]}dx$

$$\left[\because \int_0^a f(x)dx = \int_0^a f(a-x)dx\right]$$

or $I = \int_0^{\pi} \frac{(\pi - x)\tan x}{(\sec x + \tan x)}dx$ ...(*ii*)

Adding (*i*) and (*ii*), we get

$$2I = \pi . \int_0^{\pi} \frac{\tan x}{(\sec x + \tan x)}dx$$

$$= \pi \int_0^{\pi} \frac{\tan x(\sec x - \tan x)}{(\sec^2 x - \tan^2 x)}dx$$

$$= \pi\left[\int_0^{\pi} \sec x \tan x dx - \int_0^{\pi} \tan^2 x dx\right]$$

$$= \pi\left\{[\sec x]_0^{\pi} - \int_0^{\pi} (\sec^2 x - 1)\right\}dx$$

$$= \pi\left\{-2 - [\tan x]_0^{\pi} + [x]_0^{\pi}\right\} = \pi(\pi - 2)$$

$$\therefore \; I = \pi\left(\frac{\pi}{2} - 1\right), \text{ i.e., } \int_0^{\pi} \frac{x \tan x}{\sec x + \tan x}dx = \pi\left(\frac{\pi}{2} - 1\right).$$

33.

(b) We have,

$$\int_0^x f(t)dt = x + \int_x^1 t\, f(t)dt$$

$$\Rightarrow \frac{d}{dx}\left(\int_0^x f(t)dt\right) = \frac{d}{dx}\left(x + \int_x^1 tf(t)dt\right)$$

$$\Rightarrow f(x) = 1 + 0 - xf(x) \qquad \text{[Using Leibnitz's Rule]}$$

$$\Rightarrow f(x) = 1 - x\,f(x)$$

$$\Rightarrow f(x) = \frac{1}{x+1} \Rightarrow f(1) = \frac{1}{2}.$$

34.

(a) $$\int_0^{\pi/2} \log|\tan x + \cot x|dx = \int_0^{\pi/2} \log\left|\frac{\sin^2 x + \cos^2 x}{\sin x \cos x}\right|dx$$

$$= \int_0^{\pi/2} \log\left(\frac{1}{\sin x \cos x}\right)dx$$

$$= -\int_0^{\pi/2} \log \sin x\, dx - \int_0^{\pi/2} \log \cos x\, dx$$

$$= -(-\pi/2 \log 2) - (-\pi/2 \log 2) = \pi \log 2.$$

35.

(c) $$I = \int_{-\pi/2}^{\pi/2} \frac{dx}{\left(e^{\sin x} + 1\right)}$$

$$= \int_{-\pi/2}^{0} \frac{dx}{e^{\sin x} + 1} + \int_0^{\pi/2} \frac{dx}{e^{\sin x} + 1}$$

in the first integral put $x = -t \quad \therefore \quad dx = -dt$

$$= \int_{\pi/2}^{0} \frac{-dt}{e^{-\sin t} + 1} + \int_0^{\pi/2} \frac{dx}{e^{\sin x} + 1}$$

$$= \int_0^{\pi/2} \frac{dx}{e^{-\sin x} + 1} + \int_0^{\pi/2} \frac{dx}{e^{\sin x} + 1}$$

$$= \int_0^{\pi/2} 1.dx = [x]_0^{\pi/2} = \frac{\pi}{2}.$$

36.

Let $I = \int_0^{\frac{41}{2}} e^{2x-[2x]}dx$

$= \int_0^{\frac{1}{2}} e^{2x-[2x]}dx + \int_{\frac{1}{2}}^{1} e^{2x-[2x]}dx + \int_1^{\frac{3}{2}} e^{2x-[2x]}dx + \ldots$

$\int_{\frac{39}{2}}^{20} e^{2x-[2x]}dx + \int_{20}^{\frac{41}{2}} e^{2x-[2x]}dx$

$\therefore\ I = \int_0^{\frac{1}{2}} e^{2x}dx + \int_{\frac{1}{2}}^{1} e^{2x-1}dx + \int_1^{\frac{3}{2}} e^{2x-2}dx + \ldots\ldots$

$= \left[\frac{e^{2x}}{2}\right]_0^{\frac{1}{2}} + \left[\frac{e^{2x-1}}{2}\right]_{\frac{1}{2}}^{1} + \left[\frac{e^{2x-2}}{2}\right]_1^{\frac{3}{2}} + \ldots + \left[\frac{e^{2x-39}}{2}\right]_{\frac{39}{2}}^{20} + \left[\frac{e^{2x-40}}{2}\right]_{20}^{\frac{41}{2}}$

$= \left(\frac{e-1}{2}\right) + \left(\frac{e-1}{2}\right) + \left(\frac{e-1}{2}\right) + \ldots + \left(\frac{e-1}{2}\right) + \left(\frac{e-1}{2}\right)$

$= \frac{41}{2}(e-1).$

37.

(a) $\int_{-1}^{1}(x-[x])dx = \int_{-1}^{0}(x-[x])dx + \int_0^1(x-[x])dx$

$= \int_{-1}^{0}(x+1)dx + \int_0^1(x-0)dx$

$= \left[\frac{x^2}{2}+x\right]_{-1}^{0} + \left[\frac{x^2}{2}\right]_0^1 = \frac{1}{2}+\frac{1}{2} = 1.$

38.

(b) We have , $\frac{d}{dx}\left(\tan^{-1}\frac{1}{x}\right) = \frac{d}{dx}\left(\cot^{-1}x\right) = -\frac{1}{1+x^2}$

$\therefore \int_{-1}^{1}\frac{d}{dx}\left(\tan^{-1}\frac{1}{x}\right)dx = \int_{-1}^{1} -\frac{1}{1+x^2}dx$

$= -2\int_0^1 \frac{1}{1+x^2}dx = -2\left[\tan^{-1}x\right]_0^1 = -2(\pi/4) = -\pi/2$

also $\int_{-1}^{1}\frac{d}{dx}\left[\tan^{-1}\frac{1}{x}\right]dx = \left[\tan^{-1}\frac{1}{x}\right]_{-1}^{1}$

$= \tan^{-1}1 - \tan^{-1}(-1) = \frac{\pi}{2}$

is incorrect, because $\tan^{-1}\frac{1}{x}$ is not an antiderivative (primitive) of $\frac{d}{dx}\left(\tan^{-1}\frac{1}{x}\right)$ on the interval [– 1, 1].

39.

40.

(d) $\int_0^{50\pi} \sqrt{1-\cos 2x}\,dx = \int_0^{50\pi} \sqrt{2}\sin x\,dx$

$= \sqrt{2}[-\cos x]_0^{50\pi}$

$= -\sqrt{2}[\cos 50\pi - \cos 0]$

$= -\sqrt{2}[1-1] = 0$

41.

42.

(b) Since $x-[x]$ is a periodic function with period 1. Therefore, $\sin(x-[x])\pi$ is also a periodic function with period 1.

So $\int_0^{100} \sin(x-[x])\pi\,dx = 100\int_0^1 \sin(x-[x])\pi dx$

$= 100\int_0^1 \sin \pi\, x dx = -\frac{100}{\pi}[\cos \pi\, x]_0^1 = \frac{200}{\pi}$

43.

(a) Here $f(\theta) = \log\left(\frac{2-\sin\theta}{2+\sin\theta}\right)$

$\therefore \quad f(-\theta) = \log\left\{\frac{2-\sin(-\theta)}{2+\sin(-\theta)}\right\} = \log\left(\frac{2+\sin\theta}{2-\sin\theta}\right)$

$= -\log\left(\frac{2-\sin\theta}{2+\sin\theta}\right) = -f(\theta)$

$\therefore \quad I = 0.$ ($\because$ f(a) is an odd function)

44.

(a) Given $\int_0^{a-b} f(x+b)dx = k\int_b^a f(x)dx$ ...(*i*)

Putting $x+b=t$ **in first integral**

$\int_0^{a-b} f(x+b)dx = k\int_b^a f(t)dt = \int_b^a f(x)dx$ ...(*ii*)

Comparing (*ii*) with (*i*) we get

$\int_b^a f(x)dx = k\int_b^a f(x)dx$

$\Rightarrow \quad k = 1.$

45.

(a) Let $I = \int_0^{\infty} \frac{x \log x}{(1+x^2)^2} dx$

Putting $x = \tan\theta$, so that $dx = \sec^2 d\theta$. We have

$$I = \int_0^{\pi/2} \frac{\tan\theta \log\tan\theta \sec^2\theta \, d\theta}{(1+\tan^2\theta)^2}$$

$$= \int_0^{\pi/2} \sin\theta\cos\theta \log\tan\theta \, d\theta \qquad ...(i)$$

$$\Rightarrow \quad I = \int_0^{\pi/2} \sin\left(\frac{\pi}{2}-\theta\right)\cos\left(\frac{\pi}{2}-\theta\right)\log\tan\left(\frac{\pi}{2}-\theta\right)d\theta$$

$$\Rightarrow \quad I = \int_0^{\pi/2} \cos\theta\sin\theta \log\cot\theta \, d\theta \qquad ...(ii)$$

Adding (*i*) and (*ii*), we get

$$2I = \int_0^{\pi/2} \sin\theta\cos\theta(\log\tan\theta + \log\cot\theta)d\theta$$

$$= \int_0^{\pi/2} \sin\theta\cos\theta . \log 1 \, d\theta = 0 \Rightarrow I = 0.$$